RECURSION, TREE RECURSION, AND ORDERS OF GROWTH

COMPUTER SCIENCE 61A

July 4 to July 10, 2015

1 Higher Order Functions

1. Draw an environment diagram for the following code.

```
tower = 12

def sather(gate):
    def tower():
        gate = 1
        return lambda tower: tower + gate
    return tower

campanile = sather(tower)
campanile()(12)
```

2. Implement `mystery`, a function that passes the following doctests:

```
def mystery(func, n):
    """
    >>> from operator import add, mul
    >>> a = mystery(add, 3)
    >>> a(4)
    7
    >>> a(12)
    15
    >>> b = mystery(lambda x, y: x*x + y, 4)
    >>> b(5)
    21
    >>> b(7)
    23
    """
```

2 Recursion

1. What is a recursive function?

2. What are 3 important components that all recursive functions have?

3. What is a tree recursive function? How is it different from a linearly recursive function?

4. Do the following recursive functions work as intended? If not, find the bug and fix it.

```
def find_digit(number, digit):
    """Return true if the digit is included in the given
    number. Return false otherwise.

    >>> find_digit(4, 4)
    True
    >>> find_digit(4356, 4)
    True
    >>> find_digit(4356, 8)
    False
    >>> find_digit(3, 4)
    False
    """
    if number % 10 == digit:
        return True
    else:
        return find_digit(number // 10, digit)
```

```
def sum_digits(number):
    """Return the sum of all digits in a number.
    >>> sum_digits(4)
    4
    >>> sum_digits(43)
    7
    >>> sum_digits(123456789)
    45
    """
    if number < 10:
        return number
    else:
        return sum_digits(number % 10) + number // 10
```

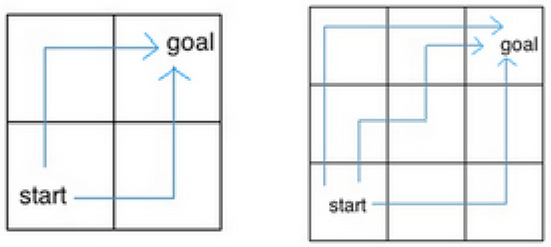
5. Implement `sorted_digits(n)`, a function that takes in a number `n` and returns True if the digits of `n` are increasing from right to left.

```
def sorted_digits(number):
    """Return True if the digit is in increasing order from
    rightmost digit to leftmost digit. (Consecutive digits
    that are the same are allowed.) Also return True if it
    has only one digit. Return False otherwise.

    >>> sorted_digits(2)
    True
    >>> sorted_digits(22222)
    True
    >>> sorted_digits(9876543210)
    True
    >>> sorted_digits(9087654321)
    False
    """
```

6. Implement `path(n)`, which returns the number of paths from one corner of an n x n grid to the opposite corner.

   Consider an insect in an N by N grid. The insect starts at the bottom left corner, (0, 0), and wants to end up at the top right corner, (N-1, N-1). The insect is only capable of moving right or up. Write a function paths that takes a grid length and width and returns the number of different paths the insect can take from the start to the goal. (There is a closed-form solution to this problem, but try to answer it procedurally using recursion.)



   For example, the 2 by 2 grid has a total of two ways for the insect to move from the start to the goal. For the 3 by 3 grid, the insect has 6 different paths (only 3 are shown above).

```
def paths(n):
    """Return the number of paths from one corner of an
    N by N grid to the opposite corner.

    >>> paths(2)
    2
    >>> paths(3)
    6
    >>> paths(10)
    48620
    """
```

3 Orders of growth

1. Write down the orders of growth for the following functions in terms of n.

```
def a(n):
    if n <= 0:
        return 1
    return 1 + a(n // 2)
```

```
def loopy(n):
    result = 0
    while n > 0:
        result += n
        n -= 1
    return result
```