Data abstraction is a powerful concept in computer science that allows programmers to treat code as objects — for example, car objects, chair objects, people objects, etc. That way, programmers don’t have to worry about how code is implemented — they just have to know what it does.

Data abstraction mimics how we think about the world. For example, when you want to drive a car, you don’t need to know how the engine was built or what kind of material the tires are made of. You just have to know how to turn the wheel and press the gas pedal.

An abstract data type consists of two types of functions:

- Constructors: functions that build the abstract data type.
- Selectors: functions that retrieve information from the data type.

For example, say we have an abstract data type called city. This city object will hold the city’s name, and its latitude and longitude. To create a city object, you’d use a constructor like

\[
\text{city} = \text{make_city(name, lat, lon)}
\]

To extract the information of a city object, you would use the selectors like

\[
\text{get_name(city)}
\]
\[
\text{get_lat(city)}
\]
\[
\text{get_lon(city)}
\]

For example, here is how we would use the make_city constructor to create a city object to represent Berkeley and the selectors to access its information.

```python
>>> berkeley = make_city('Berkeley', 122, 37)
>>> get_name(berkeley)
```
'Berkeley'

>>> get_lat(berkeley)
122

>>> get_lon(berkeley)
37

The following code will compute the distance between two city objects:

```python
from math import sqrt

def distance(city_1, city_2):
    lat_1, lon_1 = get_lat(city_1), get_lon(city_1)
    lat_2, lon_2 = get_lat(city_2), get_lon(city_2)

    return sqrt((lat_1 - lat_2)**2 + (lon_1 - lon_2)**2)
```

Notice that we don’t need to know how these functions were implemented. We are assuming that someone else has defined them for us.

It’s okay if the end user doesn’t know how functions were implemented. However, the functions still have to be defined by someone. We’ll look into defining the constructors and selectors later in this discussion.

### 1.1 Questions

1. Implement `closer_city`, a function that takes a latitude, longitude, and two cities, and returns the name of the city that is relatively closer to the provided latitude and longitude.

   You may only use selectors and constructors (introduced above) for this question. You may also use the `distance` function defined above.

```python
def closer_city(lat, lon, city1, city2):
```
2  Let’s be rational!

In lecture, we discussed the rational data type, which represents fractions with the following methods:

• \texttt{rational}(n, d) - constructs a rational number with numerator \(n\), denominator \(d\)
• \texttt{numer}(x) - returns the numerator of rational number \(x\)
• \texttt{denom}(x) - returns the denominator of rational number \(x\)

We also presented the following methods that perform operations with rational numbers:

• \texttt{add_rationals}(x, y)
• \texttt{mul_rationals}(x, y)
• \texttt{rationals_are_equal}(x, y)

There is a lot we can do with just these simple methods.

2.1  Rational Number Practice

1. Write a function that returns the given rational number \(x\) raised to positive power \(e\).

\begin{verbatim}
from math import pow

def rational_pow(x, e):
    ...
\end{verbatim}

2. Implement the following rational number methods.

\begin{verbatim}
def inverse_rational(x):
    """Returns the inverse of the given non-zero rational number""

def div_rationals(x, y):
    """Returns \(x / y\) for given rational \(x\) and non-zero rational \(y\)""
\end{verbatim}
3. The irrational number $e \approx 2.718$ can be generated from an infinite series. Let’s try calculating it using our rational number data type! The mathematical formula is as follows:

$$e = \frac{1}{0!} + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \cdots$$

Write a function `approx_e` that returns a rational number approximation of $e$ to `iter` amount of iterations. We’ve provided a factorial function.

```python
def factorial(n):
    return 1 if n == 0 else n * factorial(n - 1)

def approx_e(iter=100):
```

3 My Life for Abstraction

So far, we’ve only used data abstractions. Now let’s try creating some! In the next section, we’ll be looking at two ways of implementing abstract data types: lists and functions.

3.1 Lists, or Zerg Rush!

One way to implement abstract data types is with the Python list construct.

```python
>>> nums = [1, 2]
>>> nums[0]
1
>>> nums[1]
2
```

We use the square bracket notation to access the data we stored in `nums`. The data is zero indexed: we access the first element with `nums[0]` and the second with `nums[1]`.

Let’s now use data abstractions to recreate the popular video game Starcraft: Brood War. In Starcraft, the three races, Zerg, Protoss, and Terran, create “units” that they send to attack each other.
1. Implement the constructors and selectors for the unit data abstraction using lists. Each unit will have a string catchphrase and an integer amount of damage.

```python
def make_unit(catchphrase, damage):

def get_catchphrase(unit):

def get_damage(unit):
```

3.2 Data Abstraction Violations, or, I Long For Combat!

Data abstraction violations happen when we assume we know something about how our data is represented. For example, if we use pairs and we forget to use a selector and instead use the index.

```python
>>> raynor = make_unit('This is Jimmy.', 18)
>>> print(raynor[0])  # violation!!!!
This is Jimmy.
```

In this example, we assume that `raynor` is represented as a list because we use the square bracket indexing. However, we should have used the selector `get_catchphrase`. This is a data abstraction violation.

1. Let’s simulate a battle between units! In a battle, each unit yells its respective catchphrase, then the unit with more damage wins. Implement `battle`, which prints the catchphrases of the first and second unit in that order, then returns the unit that does more damage. The first unit wins ties. Don’t violate any data abstractions!
def battle(first, second):
    """Simulates a battle between the first and second unit
    >>> zealot = make_unit('My life for Aiur!', 16)
    >>> zergling = make_unit('GRAAHHH!', 5)
    >>> winner = battle(zergling, zealot)
    GRAAHHH!
    My life for Aiur!
    >>> winner is zealot
    True
    """
The second way of constructing abstract data types is with higher order functions. We can implement the functions `pair` and `select` to achieve the same goal.

```python
>>> def pair(x, y):
    """Return a function that represents a pair of data."""
    def get(index):
        if index == 0:
            return x
        elif index == 1:
            return y
    return get

>>> def select(p, i):
    """Return the element at index i of pair p""
    return p(i)
```

```python
>>> nums = pair(1, 2)
>>> select(nums, 0)
1
>>> select(nums, 1)
2
```

Note how although using functional pairs is different syntactically from lists, it accomplishes the exact same thing.

We can tie this in with our continuing Starcraft example. Units require resources to create, and in Starcraft, these resources are called “minerals” and “gas.”

1. Write constructors and selectors for a data abstraction that combines an integer amount of minerals and gas together into a bundle. Use functional pairs.

```python
def make_resource_bundle(minerals, gas):
    
    def get_minerals(bundle):
        
    def get_gas(bundle):
```

1. Implement the functions \texttt{max\_product}, which takes in a list and returns the maximum product that can be formed using nonconsecutive elements of the list. The input list will contain only numbers greater than or equal to 1.

\begin{verbatim}
def max_product(lst):
    """Return the maximum product that can be formed using lst without using any consecutive numbers
    >>> [10,3,1,9,2] # 10 * 9
    90
    """
\end{verbatim}
2. Draw the environment diagram for the following code:

doug = "ni"

def cat(dog):
    def rug(rat):
        doug = lambda doug: rat(doug)
        return doug
    return rug(dog)("ck")

    cat(lambda rat: doug + rat)