
CS 61A Structure and Interpretation of Computer Programs

Summer 2015

FINAL SOLUTIONS

INSTRUCTIONS

- You have 3 hours to complete the exam.
- The exam is closed book, closed notes, closed computer, closed calculator, except one hand-written 8.5" × 11" crib sheet of your own creation and the official CS 61A midterm 1, midterm 2, and final exam study guides.
- Mark your answers **on the exam itself**. We will *not* grade answers written on scratch paper.

Last name	
First name	
Student ID number	
BearFacts email	
TA	
Name of the person to your left	
Name of the person to your right	
<i>All the work on this exam is my own.</i> (please sign)	

1. (12 points) What Would Scheme Do?

Assume that you have started the interactive Scheme interpreter from Project 4 using `python3 scheme.py` and executed the following statements:

```
(define lst '(1 2 (+ 3 4)))

(define y 1)

(define f (mu (x) (+ x y)))

(define g (lambda (x y) (f (+ x x))))

(define (h z) (z 3 7))
```

For each of the expressions in the table below, write the output displayed by the interpreter when the expression is evaluated. If an error occurs, write “Error”. Assume each is executed in the same interactive session started above; **the result of executing one line may affect the result of other lines.**

Expression	Interactive Output
<code>(+ 1 2 3)</code>	6
<code>(/ 1 0)</code>	Error
<code>lst</code>	<code>(1 2 (+ 3 4))</code>
<code>(car (cdr lst))</code>	2
<code>(cons (cons 3 (cons 4 nil)) 5)</code>	<code>((3 4) . 5)</code>
<code>(or 1 (/ 1 0))</code>	1
<code>(and 1 (/ 1 0))</code>	Error
<code>(h +)</code>	10
<code>(define x 5)</code>	x
<code>(f (+ x x))</code>	11
<code>(h g)</code>	13
<code>(let ((a 1) (2 2)) a)</code>	Error
<code>(let ((x 1) (y x)) y)</code>	5
<code>(f y)</code>	2

2. (16 points) Broken promises

(a) (8 pt) Fill in the environment diagram that results from executing the code below until the entire program is finished, an error occurs, or all frames are filled. You only need to show the final state of each frame. You may not need to use all of the spaces or frames.

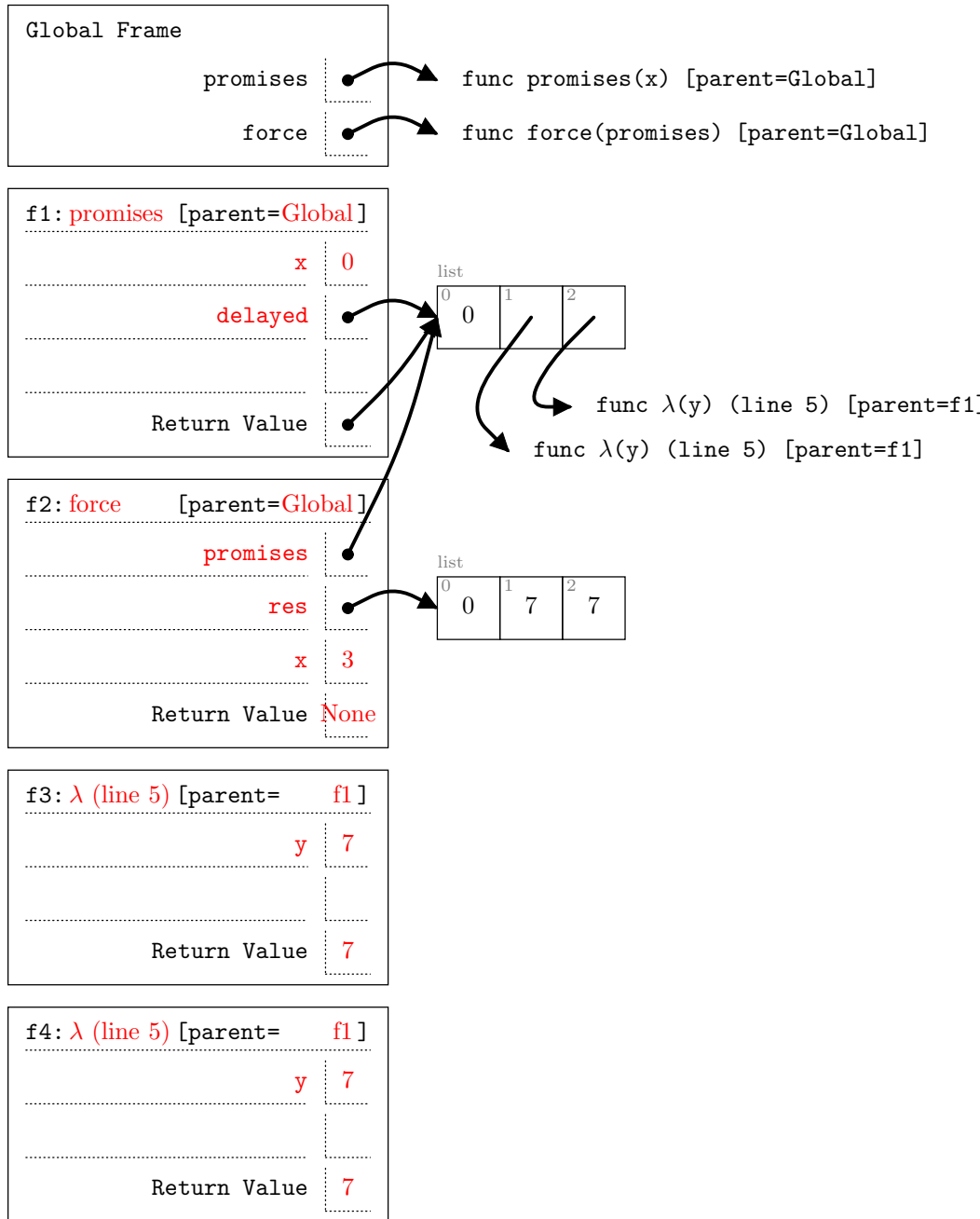
A complete answer will:

- Add all missing names and parent annotations to all local frames.
- Add all missing values created or referenced during execution.
- Show the return value for each local frame.

```

1 def promises(x):
2   delayed = [0]
3   while x > 0:
4     delayed.append(
5       lambda y: y - x)
6     x = x - 1
7   return delayed
8
9 def force(promises):
10  res = [0]
11  x = 1
12  while x < len(promises):
13    res.append(
14      promises[x](7))
15    x = x + 1
16
17 force(promises(2))

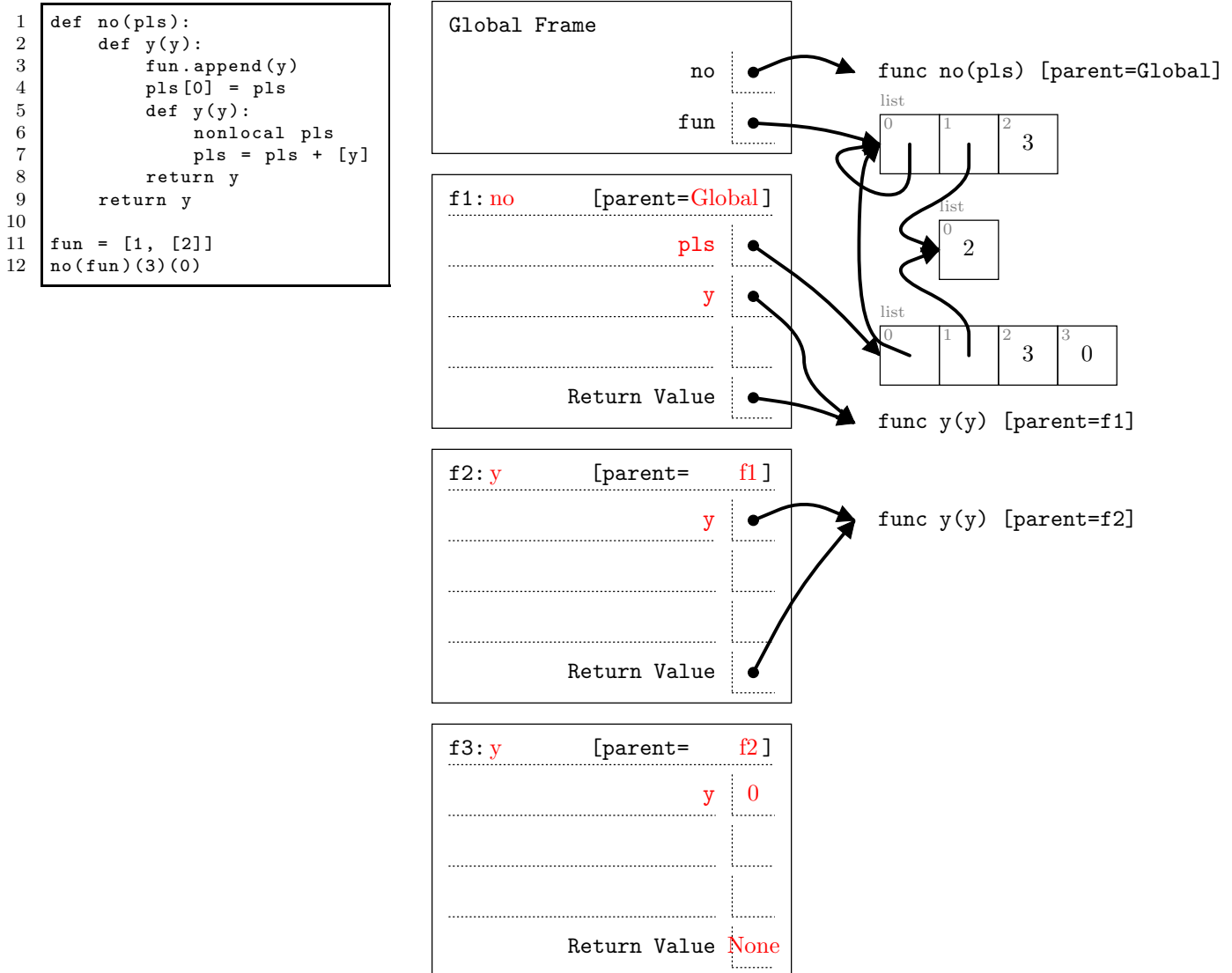
```



(b) (8 pt) Fill in the environment diagram that results from executing the code below until the entire program is finished, an error occurs, or all frames are filled. You only need to show the final state of each frame. You may not need to use all of the spaces or frames.

A complete answer will:

- Add all missing names and parent annotations to all local frames.
- Add all missing values created or referenced during execution.
- Show the return value for each local frame.



3. (4 points) Cracking the Coding Interview

The `fizzbuzz` question is a standard software engineering interview question that goes as follows:

Implement `fizzbuzz(n)`, which prints numbers from 1 to `n` (inclusive). However, for numbers divisible by 3, print “fizz”. For numbers divisible by 5, print “buzz”. For numbers divisible by both 3 and 5, print “fizzbuzz”.

A recent Reddit thread expressed concern that UC Berkeley computer science students did not know how to solve `fizzbuzz`. Prove them wrong.

```
def fizzbuzz(n):
    """
    >>> result = fizzbuzz(16)
    1
    2
    fizz
    4
    buzz
    fizz
    7
    8
    fizz
    buzz
    11
    fizz
    13
    14
    fizzbuzz
    16
    >>> result is None
    True
    """

    for i in range(1, n + 1):

        if i % 3 == 0 and i % 5 == 0:

            print('fizzbuzz')

        elif i % 3 == 0:

            print('fizz')

        elif i % 5 == 0:

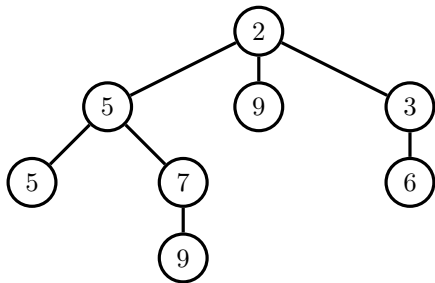
            print('buzz')

        else:

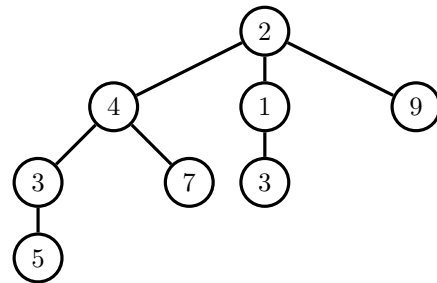
            print(i)
```

4. (8 points) I'm getting TREEtment for my heap problem

- (a) (4 pt) In computer science, a *min heap* is a tree such that every node's entry is less than or equal to all of the entries in that node's subtrees.



This is a valid min heap



Not a valid min heap. 4 is greater than 3. Also, 2 is greater than 1.

Implement `is_min_heap(t)`, where `t` is an instance of the `Tree` class (provided below). `is_min_heap` returns `True` if `t` is a valid min heap and returns `False` if it is not a valid min heap.

The `Tree` class has been provided for you below. A reasonable solution is around 6 lines, but you can use fewer or more if you want.

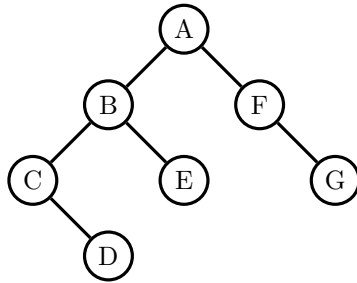
```

class Tree:
    def __init__(self, entry, subtrees=[]):
        self.entry = entry
        self.subtrees = list(subtrees)
    def is_leaf(self):
        return not self.subtrees

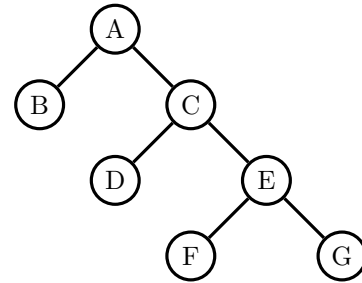
def is_min_heap(t):
    if t.is_leaf():
        return True
    for subtree in t.subtrees:
        if t.entry > subtree.entry or not is_min_heap(subtree):
            return False
    return True
  
```

(b) (4 pt)

A *full binary tree* is a `BinaryTree` where every node is either a leaf or has two children. Consider the following `BinaryTrees`:



Not a full binary tree; nodes C and F only have one child.



A full binary tree; every node is either a leaf or has two children.

Implement `is_full(b)`, where `b` is an instance of the `BinaryTree` class. `is_full` returns `True` if `b` is a full binary tree and returns `False` otherwise. You may assume `b` will not be the empty binary tree.

The `BinaryTree` class has been provided for you. A reasonable solution is around 5 to 7 lines, but you can use fewer or more if you want.

```

class BinaryTree:
    empty = ()
    def __init__(self, entry, left=empty, right=empty):
        self.entry = entry
        self.left = left
        self.right = right

def is_full(b):

    if b.left is BinaryTree.empty and b.right is BinaryTree.empty:

        return True

    elif b.left is BinaryTree.empty or b.right is BinaryTree.empty:

        return False

    else:

        return is_full(b.left) and is_full(b.right)
  
```

5. (9 points) CS61A+

(a) (4 pt) The TAs are building a social networking website called CS61A+. The TAs plan to represent the network in a class called `Network` that supports the following method:

- `add_friend(user1, user2)`: adds `user1` and `user2` to each other's friends lists. If `user1` or `user2` are not in the `Network`, add them to the dictionary of friends.

Help the TAs implement these two methods to make their social networking website popular!

```
class Network:
    """
    >>> cs61a_plus = Network()
    >>> cs61a_plus.add_friend('Robert', 'Jeffrey')
    >>> cs61a_plus.friends['Robert']
    ['Jeffrey']
    >>> cs61a_plus.friends['Jeffrey']
    ['Robert']
    >>> cs61a_plus.add_friend('Jessica', 'Robert')
    >>> cs61a_plus.friends['Robert']
    ['Jeffrey', 'Jessica']
    """
    def __init__(self):
        self.friends = {}          # Maps users to a list of their friends

    def add_friend(self, user1, user2):
        if user1 not in self.friends:
            self.friends[user1] = []
        if user2 not in self.friends:
            self.friends[user2] = []
        self.friends[user1].append(user2)
        self.friends[user2].append(user1)
```


- (b) (5 pt) CS61A+ turns out to be unpopular. To attract more users, the TAs want to implement a feature that checks if two users have at most n degrees of separation. Consider the following CS61A+ Network:

```
self.friends = {
    'Robert': ['Jeffrey', 'Jessica'],
    'Jeffrey': ['Robert', 'Jessica', 'Yulin'],
    'Jessica': ['Robert', 'Jeffrey', 'Yulin'],
    'Yulin': ['Jeffrey', 'Jessica'],
    'Albert': []
}
```

- There is 1 degree of separation between Robert and Jeffrey, because they are direct friends.
- There are 2 degrees of separation between Robert and Yulin (Robert → Jessica → Yulin)
- The degree of separation between Albert and anyone else is undefined, since Albert has no friends.

Help the TAs implement the `degrees(user1, user2, n)` method, which returns `True` if `user1` and `user2` are separated by **at most n degrees** (fewer degrees is okay). You can assume that `user1` and `user2` are already in the Network.

```
class Network:
    # Code from previous question

    def degrees(self, user1, user2, n):
        """
        >>> cs61a_plus = Network()
        >>> cs61a_plus.friends = {
        ...     'Robert': ['Jeffrey', 'Jessica'],
        ...     'Jeffrey': ['Robert', 'Jessica', 'Yulin'],
        ...     'Jessica': ['Robert', 'Jeffrey', 'Yulin'],
        ...     'Yulin': ['Jeffrey', 'Jessica'],
        ...     'Albert': []
        ... }
        >>> cs61a_plus.degrees('Robert', 'Yulin', 2)    # Exactly 2 degrees
        True
        >>> cs61a_plus.degrees('Robert', 'Jessica', 2)  # Less than 2 degrees
        True
        >>> cs61a_plus.degrees('Yulin', 'Robert', 1)    # More than 1 degree
        False
        >>> cs61a_plus.degrees('Albert', 'Jessica', 10) # No friends!
        False
        """

        if user1 == user2:

            return True

        elif n <= 0:

            return False

        for friend in self.friends[user1]:

            if self.degrees(friend, user2, n - 1):

                return True
```

```
return False
```

6. (14 points) Unique-Lo

- (a) (5 pt) Implement an iterator class called `Unique`. The `__init__` method for `Unique` takes an iterable. The `Unique` iterator represents a sequence that contains each element in the iterable only once. Only the first occurrence of each element should be included in `Unique`.

See the doctests for expected behavior. A reasonable solution is around 4 to 5 lines, but you can use fewer or more if you want.

```
class Unique:
    """
    >>> list(Unique(range(5)))          # All elements are unique
    [0, 1, 2, 3, 4]

    >>> x = [1, 2, 1, 5, 2, 5]
    >>> list(Unique(x))
    [1, 2, 5]

    >>> s = Unique([3, 3, 3, 3, 3, 3, 3, 4])
    >>> next(s)
    3
    >>> next(s)
    4
    """

    def __init__(self, iterable):

        self.iterator = iter(iterable)

        self.seen = []

    def __iter__(self):

        return self

    def __next__(self):

        value = next(self.iterator)

        while value in self.seen:

            value = next(self.iterator)

        self.seen.append(value)

        return value

    # Alternate solution

    for elem in self.iterator:

        if elem not in self.seen:
```

```
self.seen.append(elem)  
return elem
```

- (b) (4 pt) Implement a Scheme function called `contains`, which takes in a number and a well-formed Scheme list of numbers. `contains` returns True if the number appears in the list.

```
scm> (contains 3 (1 2 3 4))
True
scm> (contains 42 (3 4 1))
False
```

A reasonable solution uses 3 to 4 additional lines of code, but you can use fewer or more if you want.

```
(define (contains num lst)

  (cond ((null? lst) False)

        ((= num (car lst)) True)

        (else (contains num (cdr lst)))))
```

- (c) (5 pt) Implement a Scheme function called `unique-stream`, which takes in a stream and returns a new stream that contains each element of the input stream only once. Only the first occurrence of each element should be included, and should be included in the order in which it appears in the original stream.

Hint: you may assume `contains` function from the previous question is implemented correctly and use it in this question. You do not have to write a tail recursive function for this question.

```
scm> (define s (cons-stream 1 (cons-stream 2 (cons-stream 1
                                             (cons-stream 5 (cons-stream 2 nil)))))
scm> (define result (unique-stream s))
scm> (stream-to-list result 3)
(1 2 5)
```

A reasonable solution uses 3 to 5 additional lines of code, but you can use fewer or more if you want.

```
(define (unique-stream s)

  (define (helper s seen)

    (cond ((null? s) nil)

          ((contains (car s) seen)

           (helper (stream-cdr s) seen))

          (else (cons-stream (car s)

                              (helper (stream-cdr s) (cons (car s) seen))))))

  )

  (helper s nil)
)
```

7. (6 points) Counting stars

- (a) (4 pt) Implement a tail-recursive Scheme function called `count`, which takes in an item and a well-formed Scheme list. `count` returns the number of times the item appears in the list.

```
scm> (count 3 (1 2 3 4 3))
2
scm> (count 42 (3 4 2))
0
```

Remember, **your solution must be tail recursive**. A reasonable solution uses 3 to 4 additional lines of code, but you can use fewer or more if you want. Make sure to fill in the blanks in the second-to-last line.

```
(define (count elem lst)

  (define (helper lst total)

    (cond ((null? lst) total)

          ((= (car lst) elem) (helper (cdr lst) (+ total 1)))

          (else (helper (cdr lst)))))

  )

  (helper lst 0)

)
```

- (b) (2 pt) When the Berts eat at a restaurant, they record a review in a SQL table called `reviews`:

restaurant	user	stars	review
Barney's	Albert	4	Used to like it
Chipotle	Robert	5	BOGO! BOGO!
Eureka	Albert	5	My favorite!
Bongo Burger	Albert	2	When I'm desperate
Umami Burger	Albert	5	I love truffle fries!

Write an SQL query to figure out how many restaurants Albert gave 4 or 5 stars. Using the table above, the output to your query should be the following:

stars	number of reviews
4	1
5	2

```
select stars, count(*) from reviews

where user = "Albert"

group by stars

having stars >= 4;
```

8. (9 points) Subsequently...

- (a) (5 pt) Implement a Python function called `subsequences(lst)`, which takes in a Python list of unique numbers. `subsequences` returns a list of all subsequences of `lst`.

A list `A` is a *subsequence* of list `B` if `A` contains zero or more elements of `B` in the same order that they appear in `B`.

See the doctests for expected behavior. We will not penalize you for returning the subsequences in a different order than the doctests. However, the order of elements within a subsequence must be correct.

```
def subsequences(lst):
    """
    >>> result = subsequences([1, 2, 3])
    >>> for subsequence in result:
    ...     print(subsequence)
    []
    [3]
    [2]
    [2, 3]
    [1]
    [1, 3]
    [1, 2]
    [1, 2, 3]
    """

    if lst == []:
        return [[]]

    without_first = subsequences(lst[1:])

    with_first = [lst[0] + elem for elem in without_first]

    return without_first + with_first
```

- (b) (4 pt) In the previous question, we had to explicitly tell Python how to construct the list of subsequences, because Python is an imperative programming language.

Recall that, since Logic is a declarative programming language, we just have to describe what a subsequence looks like, and then Logic will figure out all of the subsequences for us with the following query:

```
logic> (query (subsequence (1 2 3) ?what))
what: (1 2 3)
what: (1 2)
what: (1 3)
what: (1)
what: (2 3)
what: (2)
what: (3)
what: ()
```

In this question, we will write a set of facts for the `subsequence` relation, which describes what a subsequence of a list looks like.

- (1 pt) Write a fact for the base case:

```
(fact (subsequence () ()))
```

- (2 pt) Next, write a recursive fact for the case where the first element of the list is the same as the first element in the subsequence. For example,

```
logic> (query (subsequence (1 2 3 4) (1 2 4)))
Success!
```

```
(fact (subsequence (?a . ?r1) (?a . ?r2))
      (subsequence ?r1 ?r2))
```

- (1 pt) Finally, write a recursive fact for the case where the first element of the list might not be the same as the first element in the subsequence. For example,

```
logic> (query (subsequence (1 2 3 4) (2 3)))
Success!
```

```
(fact (subsequence (?a . ?r) ?sub)
      (subsequence ?r ?sub))
```


9. (0 points) Extra lectures, extra credit

The following questions are each worth 0.5 extra credit points. Each question is from one of the special interest lectures given in the last week of class.

- (a) (0.5 pt) Machine Learning: What is the difference between a reward function and a value function?

A reward function calculates the immediate score you get from entering a state. A value function calculates the long-term score you get from starting at a state.

- (b) (0.5 pt) Computability and Complexity: What is the difference between P and NP?

P is the set of decision problems that can be solved in polynomial time. NP is the set of decision problems that can be verified in polynomial time.

- (c) (0.5 pt) Computational Biology: What is the edit distance between “sitting” and “biting”, assuming mismatches have a cost of 1?

2

10. (0 points) Third time’s the charm?

In each of the three boxes below, write a positive integer. If one of the numbers you pick is the lowest unique integer in the class, you get one extra credit point. In other words, you get three chances to write the smallest positive integer that you think no one else will write.

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11. (0 points) Drawing to a close

Thanks for a great semester! We enjoyed teaching you this summer and we hope you enjoyed the class as well. Whether or not you plan on pursuing computer science in the future, we hope you learned a lot.

Feel free to leave a quick poem, haiku, joke, or drawing in the blank space below!